

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

THIRD SEMESTER – NOVEMBER 2009

S 918 – COMPUTATIONAL STATISTICS - III

Date & Time: 10/11/2009 / 9:00 - 12:00 Dept. No.

Max. : 100 Marks

Answer ALL the Questions.

1. a). Suppose the one step transition probability matrix is as given below

Find (i) $p_{00}^{(2)}$ (ii) $f_{00}^{(n)}$ (iii) $f_{13}^{(n)}$ and (iv) $f_{33}^{(n)}$.

$$P = \begin{bmatrix} 0.4 & 0.0 & 0.6 & 0.0 \\ 0.2 & 0.4 & 0.2 & 0.2 \\ 0.5 & 0.0 & 0.5 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.5 \end{bmatrix}$$

(16 marks)

(b). For a three state Markov chain with states {0,1,2} and transition probability matrix

$$P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix},$$

find the mean recurrence times of states 0,1 and 2.

(18 marks)

(OR)

(c). Consider a Markov chain having state space {0,1,2} and transition probability matrix

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1-p & 0 & p \\ 0 & 1 & 0 \end{bmatrix}$$

(i) Show that the chain is irreducible.

(ii) Obtain the period d for this Markov chain.

(iii) Obtain $\lim_{n \rightarrow \infty} P^{dn}$

(12 marks)

(d). An infinite Markov chain on the set of non-negative integers has the transition function as follows:

$$p_{k0} = \frac{1}{k+2} \quad \text{and} \quad p_{k, k+1} = \frac{k+1}{k+2}$$

i) Find whether the chain is positive recurrent, null recurrent or transient.

ii) Find the stationary distribution, in case it exists.

(22 marks)

2. (a). It is felt that a certain drug may lead to changes in the level of 3 biochemical compounds found in the brain. 24 mice of the same type were randomly divided into 2 groups, with the second receiving periodic administration of the drug. Both samples received in the same care and diet, although 2 of the control group mice died of natural causes during the experiment. Assays of the brains of the sacrificed mice revealed these amounts of the compounds in micrograms per gram of brain tissue:

control			Drug		
X_1	X_2	X_3	X_1	X_2	X_3
1.21	0.61	0.70	1.40	0.50	0.71
0.92	0.43	0.71	1.17	0.39	0.69
0.80	0.35	0.71	1.23	0.44	0.70
0.85	0.48	0.68	1.19	0.37	0.72
0.98	0.42	0.71	1.38	0.42	0.71
1.15	0.52	0.72	1.17	0.45	0.70
1.10	0.50	0.75	1.31	0.41	0.70
1.02	0.53	0.70	1.30	0.47	0.67
1.18	0.45	0.70	1.22	0.29	0.68
1.09	0.40	0.69	1.00	0.30	0.70
			1.12	0.27	0.72
			1.09	0.35	0.73

Test the hypothesis of no difference in the control and drug mean vectors under the assumption of multivariate normal distribution with a common unknown covariance matrix.

(33 marks)

(OR)

- (b). Obtain the first two principal components and obtain the percentage of variation explained by the components based on the variance covariance matrix of three variables.

$$S = \begin{bmatrix} 2.2605 & 2.1763 & 1.6342 \\ & 2.6605 & 1.8237 \\ & & 2.4710 \end{bmatrix}$$

(33 marks)

3. (a). Use dual simplex method to solve the following LPP:

Minimize

$$z = 3X_1 + X_2$$

Subject to the constraints:

$$X_1 + X_2 \geq 1, \quad 2X_1 + 3X_2 \geq 2, \quad X_1, X_2 \geq 0.$$

(10 marks)

- (b). Solve the following 2 x 4 game graphically.

		Player B				
Player A	[2	1	0	-2]
	1	0	3	2		

(10 marks)

(c). Solve the following transportation Problem:

	D	E	F	G	Supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

(14 marks)

(d). The following table lists the jobs of a network along with their time estimates:

Job		Duration (days)		
i	j	Optimistic	Most likely	Pessimistic
1	2	3	6	15
1	6	2	5	14
2	3	6	12	30
2	4	2	5	8
3	5	5	11	17
4	5	3	6	15
6	7	3	9	27
5	8	1	4	7
7	8	4	19	28

- (i) Draw the project network.
- (ii) Calculate the length and variance of the critical path.
- (iii) what is the approximate probability that the jobs on the critical path will be completed in 45 days?
- (iv) what is the chance of the project duration exceeding 46 days? **(20 marks)**

(e). Lubecar specializes in fast automobile oil change. The garage buys car oil in bulk at \$3 per gallon. A price discount of \$2.50 per gallon is available if Lubecar purchases more than 1000 gallons. The garage services approximately 150 cars per day and each oil change requires 1.25 gallons. Lubecar stores bulk oil at the cost of \$0.02 per gallon per day. Also the cost of placing an order for bulk oil is \$20. There is a 2 day lead time for delivery. Determine the optimal inventory policy.

(14 marks)
